Constellation and Labeling Optimization for Bit-Interleaved Coded Spatial Modulation System

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***Abstract*—In spatial modulation (SM) systems, the distribution of amplitude-phase modulation (APM) symbol causes the tradeoff of detection performance between APM symbol and active antenna, which infers the overall capacity of SM system. In this paper, the APM constellation and symbol labeling of SM system is investigated. The optimized APM constellation is designed to maximize the system coded modulation average mutual information (CM-AMI). The symbol labeling of SM system is optimized with binary switching algorithm (BSA) using the symbol detection pairwise error probability (PEP) criteria. Moreover, the proposed APM constellation and symbol labeling are applied in a coded spatial modulation (SM) system with the bit-interleaved coded modulation and iterative demapping (BICM-ID). Simulation result shows the BICM-ID SM system with proposed constellation and symbol labeling has a signal-to-noise ratio (SNR) gain compared to conventional constellation system and independent demapping system.**

I. Introduction

Multiple-input Multiple-output (MIMO) technology has been widely implemented in wireless communication systems to improve capacity and stability of the wireless link. Other than spatial multiplexing and spatial diversity MIMO architecture, a new MIMO scheme named spatial modulation (SM) has been proposed recently [1], [2]. In an SM system, transmitter antennas (TAs) are selected according to the source bits to convey information. The amplitude-phase modulated (APM) symbol is transmitted through the switching active TAs. The SM receiver decodes the APM symbol and the index of active TA to get information from both the APM domain and the space domain. Since only one TA is active simultaneously, there is no inter-antenna interference (IAI) in SM system, which brings detection complexity reduction and energy efficiency improvement [3], [4]. Therefore, SM is considered as a promising MIMO technology to exploit spectrum and energy efficiency in wireless transmission.

Compared with APM transmission system such as single-input single-output (SISO) and spatial multiplexing MIMO systems, the SM system also utilize the active TA selection in the space domain. It is shown in [5] that the distribution of APM symbols causes mutual information tradeoff between the APM and space domain. This means the constellations designed for the APM system, such as quadrature amplitude modulation (QAM) and amplitude phase shift keying (APSK), are not optimal in SM system in capacity and error performance. Researches are proposed to investigate the constellation suitable for SM system. In [6], the minimized error probability of SM system is used as an objective to optimize the APM constellation, and the average symbol error rate (SER) is reduced. In [7], a multi-ring structure of constellation is considered for SM system, and an algorithm is proposed to search for the symbol distribution on the rings. In [8], the design of differential SM (DSM) with the APSK constellation is studied based on an average bit error probability (ABEP) upper bound.

The existing literatures focus on either SM constellation with certain structure constraint or the optimization of error probability. However, how the SM constellation affects the system transmission rate remains unsolved. Moreover, for a coded SM system, not only the constellation, but also symbol labeling and coded modulation (CM) scheme needs to be taken into consideration. In this paper, we propose an algorithm to design the APM constellation that optimize the CM average mutual information (CM-AMI) of SM system. The symbol labeling that determines how the bit vectors map to the symbols in SM system is optimized with the symbol detection pairwise error probability (PEP) using the binary switch algorithm (BSA) [9].

Recent researches mainly aimed to improve the symbol and bit error performance of uncoded SM system. In practical systems, channel coding is used for error correction and to ensure quasi error-free transmission above signal-to-noise ratio (SNR) threshold. Therefore, it is important to study SM with channel coding. The bit-interleaved coded modulation (BICM) is an efficient CM scheme with low complexity [10]. With ideal bit interleaver, each bit in BICM system can be regarded as transmitted through parallel additive white Gaussian noise (AWGN) channel. Combined with iterative demapping [11], the performance loss of the traditional independent demapping can be mitigated and the CM-AMI limit can be approached with the iteration process. Therefore, in this paper, we adopt BICM-ID scheme to design coded SM system.

The rest parts of the paper are organized as follows. In Section II, we present the system model of SM system with BICM-ID. In Section III, we introduce an APM constellation design algorithm for SM system to optimize the CM-AMI. The constellation labeling optimization is considered in Section IV. The numerical simulation is used in Section V to calculate the proposed constellation and symbol labeling, of which the performance is also evaluated. Finally, Section VI concludes the whole paper.

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II. System Model

A. Spatial Modulation

We consider an SM system with N_t TAs, N_r receiver antennas (RAs) and an N_s-ary finite constellation. The transmission equation of SM system is given by

$$\boldsymbol{y} = \boldsymbol{h}_k \sqrt{\gamma} s_i + \boldsymbol{z} \tag{1}$$

where γ stands for the SNR, $s \in \chi_a$ is the APM symbol and $k \in \chi_c$ is the index of activated TA. The APM constellation is denoted by $\chi_a = \{s_1, s_2, \cdots, s_{N_s}\} \subseteq \mathbb{C}$, which includes N_s discrete complex symbols. χ_c is the set of indices of activated TAs, i.e. $\chi_c = \{1 \leqslant k \leqslant N_t\}$. Thus, the joint APM and space constellation of the SM system is $\chi = \chi_a \times \chi_c$, and the number of total symbols is $M = |\chi| = N_t N_s$. $\boldsymbol{z}$ is the AWGN with unit power, i.e. $\boldsymbol{z} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_r})$. The CM-AMI of the SM system is formulated as

$$I(X, X_{\text{ch}}; \boldsymbol{Y}) = h(\boldsymbol{Y}) - h(\boldsymbol{Y}|X, X_{\text{ch}}) = h(\boldsymbol{Y}) - h(\boldsymbol{Z}), \tag{2}$$

where $X, X_{\text{ch}}, \boldsymbol{Y}$ and $\boldsymbol{Z}$ represent the random variables of $s_i, k, \boldsymbol{y}$ and $\boldsymbol{z}$, respectively. In Eqn. (2), the differential entropy $h(\boldsymbol{Z})$ can be easily derived as $h(\boldsymbol{Z}) = N_r \log_2(\pi e)$, since $\boldsymbol{Z}$ is an independent and identical distributed (i.i.d.) Gaussian random vector. However, the closed-form differential entropy $h(\boldsymbol{Y})$ is difficult to obtain. We consider the probability density function (PDF) of $\boldsymbol{Y}$ with given CSI, i.e.

$$f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) = \frac{1}{M} \sum_{k=1}^{N_t} \sum_{i=1}^{N_s} \frac{1}{\pi^{N_r}} \exp\Big(- \|\boldsymbol{y} - \boldsymbol{h}_k s_i\|^2 \Big), \tag{3}$$

where the equiprobable distribution of X_{ch} is assumed. The PDF of $\boldsymbol{Y}$ yields

$$I(X, X_{\text{ch}}; \boldsymbol{Y}|\boldsymbol{H}) = - \int_{\mathbb{C}^N} f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \log_2 f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) - N_r \log_2(\pi e). \tag{4}$$

The closed-form integral in Eqn. (4) is difficult to derive and can be computed with numerical method. For ergodic channel, the CM-AMI of SM system is formulated as

$$I(X, X_{\text{ch}}; \boldsymbol{Y}) = -\mathbb{E}_{\boldsymbol{H}} \Bigg\{ \int_{\mathbb{C}^N} f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \times \log_2 f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \mathrm{d}\boldsymbol{y} \Bigg\} - N_r \log_2(\pi e). \tag{5}$$

B. SM system with BICM-ID

The BICM-ID is considered as a coded modulation architecture with low complexity and high efficiency [10], and also provides performance that approaches the CM-AMI. Thus, we apply the BICM-ID scheme in the SM system.

Each SM symbol is mapped from $N_{\text{bpcu}} = m = \log_2 M$ bits. Denote the a priori logarithm likelihood ratio (LLR) of the ith bit as L_i^{a}, and $\boldsymbol{L}^{\text{a}} = [L_1^{\text{a}}, L_2^{\text{a}}, \cdots, L_m^{\text{a}}]^{\text{T}}$. With the interleaving process in BICM, the transmission of different bits can be regarded as independent. Thus, the a priori probability of an SM symbol is

$$\Pr(s, k|\boldsymbol{L}^{\text{a}}) = \prod_{i=1}^{m} \Pr(b_i|L_i^{\text{a}}) \tag{6}$$

where b_i is the ith bit that mapped to SM symbol (s, k), and $\Pr(b_i|L_i^{\text{a}})$ is the a priori probability of b_i, i.e.

$$\Pr(b_i|L_i^{\text{a}}) = \begin{cases} \dfrac{\exp(L_i^{\text{a}})}{1 + \exp(L_i^{\text{a}})}, & b_i = 0 \\ \dfrac{1}{1 + \exp(L_i^{\text{a}})}, & b_i = 1 \end{cases} \tag{7}$$

With the a priori probability, the iterative demapping process of SM with BICM-ID is formulated as

$$L_i^{\text{ex}} = \log \frac{\sum_{(s,k) \in \chi_i^{(0)}} f_{\boldsymbol{Y}|X, X_{\text{ch}}, \boldsymbol{H}}(\boldsymbol{y}|s, k, \boldsymbol{H}) \Pr(s, k|\boldsymbol{L}^{\text{a}})}{\sum_{(s,k) \in \chi_i^{(1)}} f_{\boldsymbol{Y}|X, X_{\text{ch}}, \boldsymbol{H}}(\boldsymbol{y}|s, k, \boldsymbol{H}) \Pr(s, k|\boldsymbol{L}^{\text{a}})} - L_i^{\text{a}} \tag{8}$$

where $\boldsymbol{L}_i^{\text{ex}} = [L_1^{\text{ex}}, L_2^{\text{ex}}, \cdots, L_m^{\text{ex}}]^{\text{T}}$ describes the extrinsic information after demapping, and is fed into the channel decoder to calculate the a priori information for the next iteration. In the first iteration, $\boldsymbol{L}^{\text{a}}$ is considered as $\boldsymbol{0}$, and the demapping is equivalent as the independent demapping.

III. APM Constellation Optimization

For APM systems, symbols from finite constellations lead to capacity shaping loss, since the optimized channel input that achieves channel capacity is continuous Gaussian distributed. This also happens in SM systems, although the Gaussian distributed APM input may not be optimal [5], which means constellations designed for APM systems with low shaping loss may not have optimal performance in SM systems. Therefore, a novel APM constellation that has minimum shaping loss for SM system needs to be designed. The CM-AMI of SM system given by Eqn. (5) is considered as the criterion for SM constellation design, since both APM and space domain mutual information are included. We regard the CM-AMI of SM over ergodic Rayleigh channel as a function with respect to the APM constellation χ_a. Thus, the cost function is defined as

$$J(\chi_a) = \mathbb{E}_{\boldsymbol{H}} \Bigg\{ \int_{\mathbb{C}^{N_r}} f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \log_2 f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \mathrm{d}\boldsymbol{y} \Bigg\}. \tag{9}$$

Therefore optimized APM constellation is given by

$$\hat{\chi_a} = \underset{\chi}{\operatorname{argmin}}\, J(\chi_a), \quad \text{s.t.} \sum_{s_i \in \chi_a} |s_i|^2 = 1. \tag{10}$$

where the optimization problem is subjected to the average power constraint.

We use gradient descent algorithm to solve the optimization problem given by (10). The gradient of cost function $J(\chi_a)$ with respect to χ_a is formulated as

$$\nabla J(\chi_a) = \left[\frac{\partial J(\chi_a)}{\partial s_1}, \frac{\partial J(\chi_a)}{\partial s_2}, \cdots, \frac{\partial J(\chi_a)}{\partial s_{N_s}} \right]^{\text{T}}, \ 1 \leqslant i \leqslant N_s \tag{11}$$

For an arbitrary symbol s_i in the APM constellation χ_a, consider the partial derivatives of $J(\chi_a)$. By switching the order derivative, mathematical expectation and integral, we have

$$\frac{\partial J(\chi_a)}{\partial s_i} = \frac{1}{\partial s_i}\mathbb{E}_{\boldsymbol{H}}\left\{ \int_{\mathbb{C}^{N_r}} f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \log_2 f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \mathrm{d}\boldsymbol{y} \right\}$$
$$= \mathbb{E}_{\boldsymbol{H}}\left\{ \int_{\mathbb{C}^{N_r}} \frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial s_i} \log_2 \left[e f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \right] \mathrm{d}\boldsymbol{y} \right\} \quad (12)$$

Consider the partial derivative of $f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})$ with respect to s_i. We denote s_i as the sum of its real and image part, i.e. $s_i = a_i + \mathrm{j}b_i$, which yields

$$\frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial a_i}$$
$$= \frac{2}{M}\sum_{k=1}^{N_t}\left\{ \frac{1}{\pi^{N_r}} e^{-\|\boldsymbol{y}-\boldsymbol{h}_k\sqrt{\gamma}s_i\|^2} \left[\|\boldsymbol{h}_k\|^2 \gamma a_i - \Re(\boldsymbol{h}_k^{\mathcal{H}}\boldsymbol{y})\sqrt{\gamma} \right] \right\}$$
$$= -\frac{2}{N_s}\,\mathbb{E}_{X_{\mathrm{ch}}}\Bigg\{ f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|s_i,k,\boldsymbol{H}) \times \left[\|\boldsymbol{h}_k\|^2 \gamma a_i - \Re(\boldsymbol{h}_k^{\mathcal{H}}\boldsymbol{y})\sqrt{\gamma} \right] \Bigg\}.$$

Similar for the image part b_i,

$$\frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial b_i} = -\frac{2}{N_s}\,\mathbb{E}_{X_{\mathrm{ch}}}\Bigg\{ f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|s_i,k,\boldsymbol{H}) \times \left[\|\boldsymbol{h}_k\|^2 \gamma b_i - \Im(\boldsymbol{h}_k^{\mathcal{H}}\boldsymbol{y})\sqrt{\gamma} \right] \Bigg\}.$$

Therefore,

$$\frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial s_i} = \frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial a_i} + \mathrm{j}\frac{\partial f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})}{\partial b_i}$$
$$= -\frac{2}{N_s}\,\mathbb{E}_{X_{\mathrm{ch}}}\Bigg\{ f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|s_i,k,\boldsymbol{H}) \times \left(\|\boldsymbol{h}_k\|^2 \gamma s_i - \boldsymbol{h}_k^{\mathcal{H}}\boldsymbol{y}\sqrt{\gamma} \right) \Bigg\}. \quad (13)$$

We plug Eqn. (13) into Eqn. (12). By regarding $f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|s_i,k,\boldsymbol{H})$ as the PDF, the integral in Eqn. (12) can be transformed into mathematical expectation as

$$\frac{\partial J(\chi_a)}{\partial s_i} = -\frac{2}{N_s}\mathbb{E}_{\boldsymbol{H},X_{\mathrm{ch}},(\boldsymbol{Y}|X=s_i,X_{\mathrm{ch}}=k)}\Bigg\{ \left(\|\boldsymbol{h}_k\|^2 \gamma s_i - \boldsymbol{h}_k^{\mathcal{H}}\boldsymbol{y}\sqrt{\gamma} \right) \log_2 \left[e f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \right] \Bigg\}, \quad (14)$$

where the expectation with respect to $\boldsymbol{Y}$ is under condition $X = s_i$ and $X_{\mathrm{ch}} = k$. In such condition, we have $\boldsymbol{y} = \boldsymbol{h}_k\sqrt{\gamma}s_i + \boldsymbol{z}$, which yeilds

$$\frac{\partial J(\chi_a)}{\partial s_i} = -\frac{2}{N_s}\mathbb{E}_{\boldsymbol{H},X_{\mathrm{ch}},\boldsymbol{Z}|X=s_i}\left\{ \boldsymbol{h}_{X_{\mathrm{ch}}}^{\mathcal{H}}\boldsymbol{z} \log_2 \left[e f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H}) \right] \right\}. \quad (15)$$

Eqn. (15) can be easily calculated with Monte Carlo method. For each $X = s_i$, randomly generate parameter tuple $(\boldsymbol{H}, X_{\mathrm{ch}}, \boldsymbol{Z})$ and calculate the value of $\boldsymbol{h}_{X_{\mathrm{ch}}}^{\mathcal{H}}\boldsymbol{z}\log_2 e f_{\boldsymbol{Y}|\boldsymbol{H}}(\boldsymbol{y}|\boldsymbol{H})$. By averaging these values, the partial derivative of $J(\chi_a)$ with respect to s_i is approximately obtained.

With the gradient of cost function $\nabla J(\chi_a)$, the numerical gradient descent algorithm can be used to solve the optimization problem with iteration process. In each iteration, the APM constellation χ_a is update along with the gradient direction by a small step α, until the cost function stops decreasing. Since χ_a is subjected to the average power constraint, the power of χ_a needs to be normalized after each update. Thus the iteration optimization process is formulated as

$$\tilde{\chi}_a(n+1) = \chi_a(n) - \alpha\nabla J(\chi_a(n))$$
$$\chi_a(n+1) = \tilde{\chi}_a(n+1) \Bigg/ \sqrt{\frac{1}{N_s}\sum_{s\in\tilde{\chi}_a(n+1)} |s|^2}.$$

IV. Labeling Optimization

A. Symbol Detection Pairwise Error Probability

In a coded communication system, the BER performance is strongly depending on the labeling of symbols, i.e. the mapping from bit sets to symbols in the constellation. The PEP was commonly used as the cost function to optimize the symbol mapping. We consider the derivation of symbol detection PEP in SM system. With given CSI matrix $\boldsymbol{H}$, the maximum likelihood (ML) pairwise symbol detection discriminant is

$$f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|s,k,\boldsymbol{H}) \underset{<_{\hat{X}=\hat{s},\hat{X}_{\mathrm{ch}}=\hat{k}}}{\overset{\geqslant_{\hat{X}=s,\hat{X}_{\mathrm{ch}}=k}}{}} f_{\boldsymbol{Y}|X,X_{\mathrm{ch}},\boldsymbol{H}}(\boldsymbol{y}|\hat{s},\hat{k},\boldsymbol{H})$$
$$\Leftrightarrow \quad \left\|\boldsymbol{y} - \boldsymbol{h}_k\sqrt{\gamma}s\right\| \underset{>_{\hat{X}=\hat{s},\hat{X}_{\mathrm{ch}}=\hat{k}}}{\overset{\leqslant_{\hat{X}=s,\hat{X}_{\mathrm{ch}}=k}}{}} \left\|\boldsymbol{y} - \boldsymbol{h}_{\hat{k}}\sqrt{\gamma}\hat{s}\right\|. \quad (16)$$

Therefore, the symbol detection PEP of SM system $\Pr[(k,s)\to(\hat{k},\hat{s})|\boldsymbol{H}]$ is derived as

$$\Pr[(k,s)\to(\hat{k},\hat{s})|\boldsymbol{H}]$$
$$= \Pr\left(\|\boldsymbol{y} - \boldsymbol{h}_k\sqrt{\gamma}s\| > \|\boldsymbol{y} - \boldsymbol{h}_{\hat{k}}\sqrt{\gamma}\hat{s}\| \,\middle|\, \boldsymbol{y} = \boldsymbol{h}_k\sqrt{\gamma}s + \boldsymbol{z}, \boldsymbol{H} \right)$$
$$= \Pr\left(\|\boldsymbol{z}\| > \|\boldsymbol{h}_k\sqrt{\gamma}s - \boldsymbol{h}_{\hat{k}}\sqrt{\gamma}\hat{s} + \boldsymbol{z}\| \,\middle|\, \boldsymbol{H} \right)$$
$$= Q\left(\sqrt{\frac{\gamma}{2}} \left\|\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}\right\| \right) \quad (17)$$

where $Q(\cdot)$ is the complementary cumulative distribution function (CCDF) of stand Gaussian random variable, i.e.

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} \exp\left(-\frac{u^2}{2} \right) \mathrm{d}u$$

In ergodic channel, the PEP of SM system is Eqn. (17) taken mathematical expectation with respect to $\boldsymbol{H}$. We assume Rayleigh fading channel, i.e. $[\boldsymbol{h}_1^{\mathrm{T}}, \boldsymbol{h}_2^{\mathrm{T}}, \cdots, \boldsymbol{h}_{N_t}^{\mathrm{T}}]^{\mathrm{T}} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_tN_r})$. $\|\boldsymbol{h}_k\|^2$ is χ^2 distributed with $2N_r$ degree-of-freedom (DoF), denoted by $h \sim \chi^2(2N_r)$. When the two SM symbols share the same active TA, i.e. $k = \hat{k}$, we have $\|\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}\|^2 = |s-\hat{s}|^2 h$. On the other hand when $k \neq \hat{k}$, since $\boldsymbol{h}_k$ and $\boldsymbol{h}_{\hat{k}}$ are independent to each other, we

have $(\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}) \sim \mathcal{CN}[\mathbf{0}, (|s|^2 + |\hat{s}|^2)\boldsymbol{I}_{N_r}]$, which means $\|\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}\|^2$ is also χ^2 distributed. Therefore, we can define $\|\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}\|^2 = d_{k,s,\hat{k},\hat{s}} h$, in which

$$d_{k,s,\hat{k},\hat{s}} = \begin{cases} |s-\hat{s}|^2, & (k=\hat{k}) \\ |s|^2 + |\hat{s}|^2, & (k \neq \hat{k}). \end{cases}$$

By substituting $d_{k,s,\hat{k},\hat{s}} h$ for $\|\boldsymbol{h}_k s - \boldsymbol{h}_{\hat{k}}\hat{s}\|^2$ in Eqn. (17), the symbol detection PEP of SM system over ergodic Rayleigh channel is derived as

$$\begin{aligned}\Pr[(k,s) \to (\hat{k},\hat{s})] &= \mathbb{E}_{\boldsymbol{H}}\Big\{Q\Big(\sqrt{\gamma h d_{k,s,\hat{k},\hat{s}}/2}\Big)\Big|\boldsymbol{H}\Big\} \\ &= \int_0^{\infty}\int_{\sqrt{\gamma h d_{k,s,\hat{k},\hat{s}}/2}}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\Big(-u^2/2\Big)\mathrm{d}u \; f_h(h)\mathrm{d}h,\end{aligned}$$

where $f_h(h)$ is the PDF of χ^2 distributed random variable h. Let $t = u^2/2$ and switch the two integral, then we have

$$\Pr[(k,s) \to (\hat{k},\hat{s})] = \int_0^{\infty} F_h\left(\frac{4t}{\gamma d_{k,s,\hat{k},\hat{s}}}\right)\exp(-t)\mathrm{d}t, \quad (18)$$

where $F_h(h)$ is the cumulative distribution function (CDF) of h, i.e.

$$F_h(h) = 1 - e^{-h}\sum_{i=0}^{N_r-1}\frac{1}{i!}h^i, \quad h > 0. \quad (19)$$

By plugging $F_h(h)$ into Eqn. (18), the PEP of SM over ergodic channel is obtained as

$$\begin{aligned}&\Pr[(k,s) \to (\hat{k},\hat{s})] = \\ &\qquad \frac{1}{2} - \frac{1}{2}\sum_{i=0}^{N_r-1}\frac{\Gamma(i+1/2)}{i!\sqrt{\pi}}\frac{4^i\sqrt{d_{k,s,\hat{k},\hat{s}}\gamma}}{(d_{k,s,\hat{k},\hat{s}}\gamma+4)^{i+1/2}}, \quad (20)\end{aligned}$$

where $\Gamma(x) = \int_0^{+\infty} t^{x-1}e^{-t}\mathrm{d}t$ stands for the Gamma function.

The PEP performance of SM system can be further examined when the SNR is high using asymptotic expansion method. We perform Taylor expansion on the sum item in Eqn. (20) at $\gamma = \infty$, i.e.

$$\begin{aligned}&\frac{4^i\sqrt{d_{k,s,\hat{k},\hat{s}}\gamma}}{(d_{k,s,\hat{k},\hat{s}}\gamma+4)^{i+1/2}} = \\ &\qquad \frac{1}{(1+d_{k,s,\hat{k},\hat{s}}\gamma/4)^i}\left[1 - \frac{2}{d_{k,s,\hat{k},\hat{s}}\gamma} + \mathcal{O}(1/\gamma)\right] \quad (21)\end{aligned}$$

where $\mathcal{O}(\cdot)$ stands for the high order infinitesimal. Plugging the expansion into Eqn. (20), when $N_r = 1$, we have

$$\begin{aligned}\Pr[(k,s) \to (\hat{k},\hat{s})] &= \frac{1}{2} - \frac{\Gamma(1/2)}{2\sqrt{\pi}}\left[1 - \frac{2}{d_{k,s,\hat{k},\hat{s}}\gamma} + \mathcal{O}(1/\gamma)\right] \\ &= \frac{1}{d_{k,s,\hat{k},\hat{s}}\gamma} + \mathcal{O}(1/\gamma) \quad (22)\end{aligned}$$

Similarly, for system with $N_r > 1$, we have

$$\Pr[(k,s) \to (\hat{k},\hat{s})] = \frac{1}{d_{k,s,\hat{k},\hat{s}}\gamma} - \frac{1}{d_{k,s,\hat{k},\hat{s}}\gamma + 4} + \mathcal{O}(1/\gamma) \quad (23)$$

Both Eqn. (22) and (23) show that when the SNR is high, the PEP of SM system is inversely proportional to $d_{k,s,\hat{k},\hat{s}}$ and γ. As mentioned above, for a pair of symbols with the same active TA, $d_{k,s,\hat{k},\hat{s}}$ is Euclidean distance of APM symbols on the complex plane; for different TAs, $d_{k,s,\hat{k},\hat{s}}$ equals to the power sum of a pair of APM symbols. This means for the APM constellation and symbol mapping of SM system, both Euclidean distance between pairs of symbols and the power of each single symbol need to be optimized.

B. Binary Switching Algorithm

With the symbol detection PEP derived above, we use the binary switching algorithm (BSA) [9] for the labeling optimization of SM system. BSA is a greedy algorithm to search for the optimal symbol mapping in BICM-ID system. In each iteration of the searching process, two labeling indices of symbols are switched with each other, and the value of a cost function is calculated. If the switching generates a lower cost than the cost before labeling switching, the switch is accepted, otherwise the switch is ignored. During the searching process, the cost keep decreasing until the presumed optimum.

For SM system, the average active TA detection symbol error rate (SER) union bound can be calculated with the active TA detection PEP, i.e.

$$\hat{D} = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{(k,s)\in\chi_i^b}\sum_{(\hat{k},\hat{s})\in\chi_i^{\bar{b}}}\Pr[(k,s) \to (\hat{k},\hat{s})], \quad (24)$$

where χ_i^b stands for a subset of χ in which the ith bit of the symbols is b. By substituting the high SNR asymptotic PEP into Eqn. (24), we have

$$D = \frac{1}{m2^m}\sum_{i=1}^{m}\sum_{b=0}^{1}\sum_{(k,s)\in\chi_i^b}\sum_{(\hat{k},\hat{s})\in\chi_i^{\bar{b}}}\frac{1}{d_{k,s,\hat{k},\hat{s}}}, \quad (25)$$

where the SNR item is ignored since it is independent to the constellation and labeling.

In [9], the cost function without a priori information and with perfect a priori information are denoted by D_0 and D_1. The a priori information of the cost function is inferred by the constellation subset χ_i^b and $\chi_i^{\bar{b}}$. When without a priori information, we have $\chi_i^b \cap \chi_i^{\bar{b}} = \chi$, while with perfect a priori information, since all other bits are known, both χ_i^b and $\chi_i^{\bar{b}}$ only contain one symbol. A weighted average cost function is used in [9] to take the two situation into consideration, i.e.

$$D_c = \lambda D_0 + (1-\lambda)D_1. \quad (26)$$

V. Numerical Simulation

In this section, we use numerical simulation to solve the constellation and labeling optimization problem of SM system presented in Section III-IV, and evaluate the performance of BICM-ID SM system. During the simulation, flat fading Rayleigh ergodic MIMO channel is assumed.

First, we investigate the SM constellation optimization problem by solving (10) with the algorithm presented in Section III. The 64-ary constellation is considered. At the

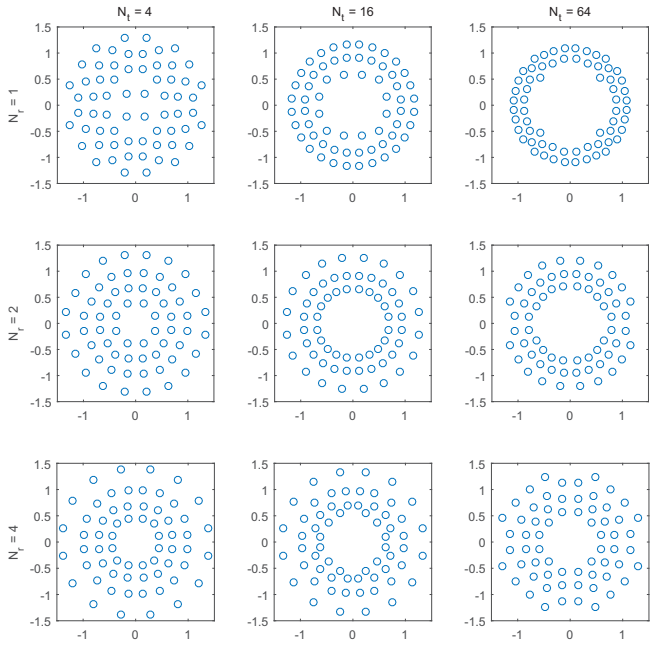
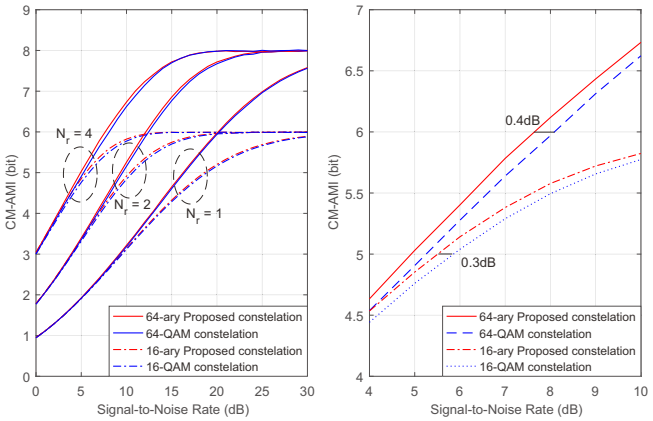


Fig. 1. The proposed CM-AMI optimized 64-ary SM constellation.



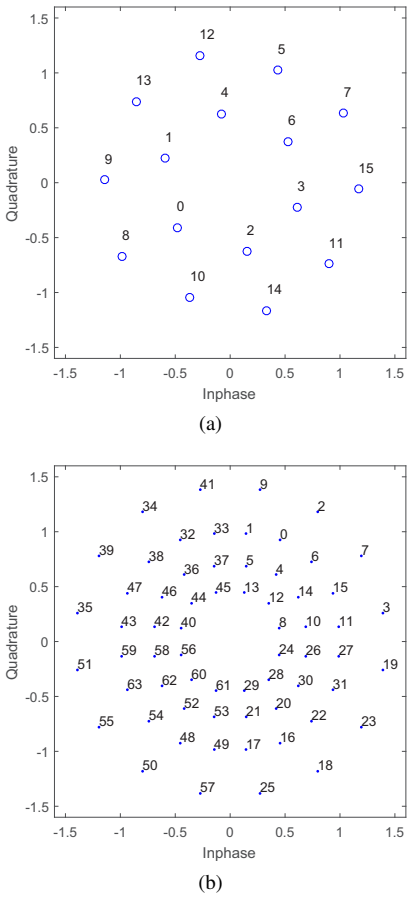
(a) (b)

Fig. 2. The CM-AMI comparison between the proposed optimized constellation and QAM. In (a), $N_t = 4$. In (b), $N_t = N_r = 4$.

beginning, a random constellation is generated, and then the constellation is update with the gradient descent algorithm until the cost function stops decreasing. The constellation symmetry constraint is used to simplify the calculation, where for $1 \leqslant i \leqslant N_s/4$, let

$$s_{i+N_s/4} = s_i^*, \quad s_{i+N_s/2} = -s_i, \quad s_{i+3N_s/4} = -s_i^*.$$

The calculated optimal SM constellation is presented in Figure 1. As can be seen from Figure 1, the proposed CM-AMI optimized constellation for SM system is different from the constellations designed for APM system, such as QAM or APSK. In Figure 1, we investigate how the numbers of TAs and RAs affect the optimal constellation of SM system. For SM system with $N_t \gg N_r$, the CM-AMI optimal constellation



(a) (b)

Fig. 3. The proposed labeling result of (a) 16-ary and (b) 64-ary SM constellation. $N_t = N_r = 4$.

resembles the phase shift keying (PSK), as shown by the $N_t = 64, N_r = 1$ case. This is because the symbols in the PSK constellation have identical power, which benefits the active TA detection. It is also shown by Eqn. (20) that for symbols with different active TAs, the PEP depents on the total power of the APM symbols. For SM system with large number of TAs, the TA detection dominates the overall SER. On the other hand, for SM system with less TAs and more RAs, Figure 1 shows the optimal constellation is more like the APSK, since in this case the APM symbol detection matters more and the APSK is optimized for APM transmission.

Figure 2 shows the CM-AMI comparison between the traditional QAM and the proposed optimized constellation. The proposed constellation has greater CM-AMI than QAM in SM system, which means by constellation optimization, the shaping loss is reduced. Figure 2b shows that for 4×4 SM-MIMO system, 64-ary proposed constellation has around 0.4dB CM-AMI shaping gain, and for 16-ary constellation the shaping gain is around 0.3dB.

Figure 3 shows the labeling results of the proposed SM constellation. In the BSA process, the weight λ in the cost function (26) is set as 1 to optimize the situation without a priori knowledge. The obtained constellation and labeling

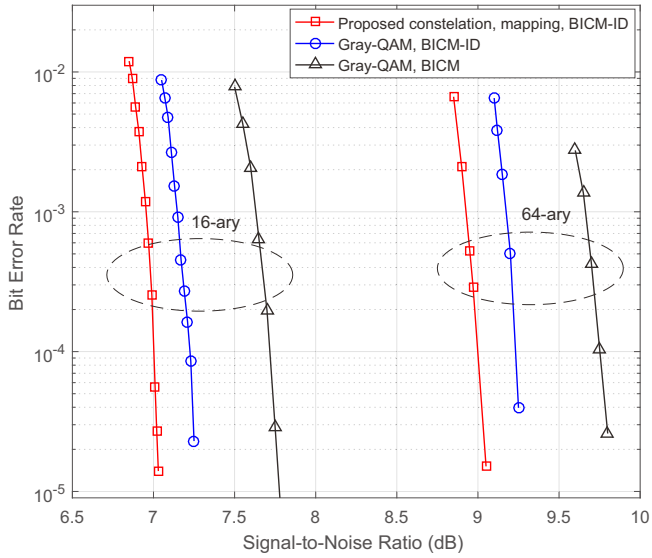


Fig. 4. BER of SM BICM-ID system with 64800 DVB-T2 LDPC. $N_t = N_r = 4$. The code rate for 16-ary is $5/6$, for 64-ary is $3/4$.

scheme is then used in the BICM-ID SM system. In the BICM system, we use the S-random interleaver where $S = 100$. The DVB-T2 low density parity check (LDPC) code with length of 64800 is used. For 16-ary constellation, the code rate is set as $5/6$, and the code rate for 64-ary constellation is $3/4$. The LDPC is iteratively decoded for 30 times, and the number of demapping iteration is 50. The bit error rate (BER) is averaged over 5000 LDPC frames. Figure 4 shows the BER result of SM BICM-ID system. As we can see from the figure, the proposed SM constellation and labeling scheme has 0.2-0.3dB of SNR gain compared with Gray mapping QAM system. The BICM-ID scheme also improves the BER performance of coded SM system. Compared with BICM SM system with Gray-QAM, the BICM-ID SM with optimal constellation and labeling shows a 0.7-0.8dB SNR gain.

VI. Conclusions

In this paper, we study the constellation and labeling optimization problem of SM system. A novel SM constellation design method based on CM-AMI and an SM labeling design method based on BSA is proposed. The CM-AMI of SM system is used as the cost function to optimized the constellation with gradient descent algorithm. The symbol detection PEP of SM system is derived and used as the cost function for optimal SM labeling searching. We combine the BICM-ID architecture with the SM system, to realize performance that approaches the CM-AMI. Numerical simulation shows that our proposed SM constellation provides CM-AMI shaping gain to the SM system. The proposed constellation and labeling, applied in SM system with BICM-ID, has better BER performance than the traditional constellation and independent demapping system.

VII. Acknowledgement

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